Chaos, Complexity and Neural Network Time Series Predictions

Sebastian Raubitzek, University of Technology, Vienna.

I want you to consider reconstructed phase spaces when analyzing or predicting time series.

About this Talk

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- New ideas on how to use reconstructed phase spaces, basic concepts.

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- Phase space reconstruction and time series interpolation.
- Phase space reconstruction and time series prediction.
- This talk is not focused on experimental results.

Prerequisites

What are reconstructed phase spaces?

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- Reconstructing multidimensional dynamics from univariate time series

What are reconstructed phase spaces?

- Reconstructing multidimensional dynamics from univariate time series
- Phase space: Underlying structure, Symmetry, regularities

How does it work?

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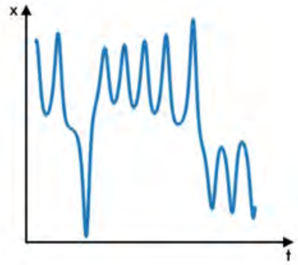

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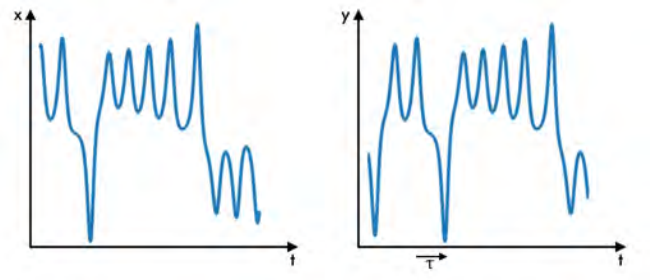

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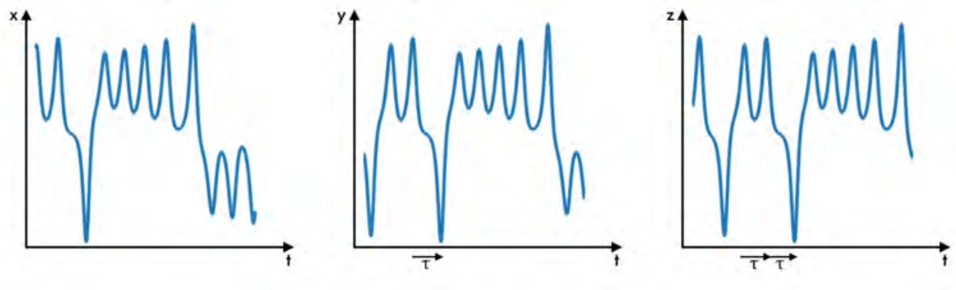

Figure: *Phase space embedding, i.e. depiction of the time delay.*

Example - The Lorenz System

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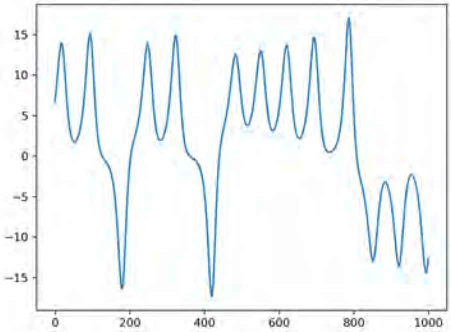

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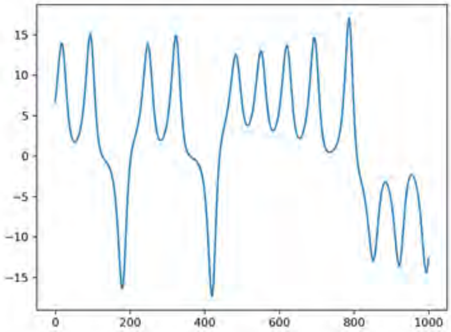

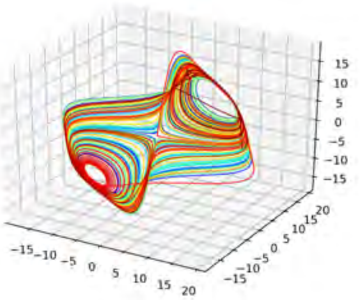

Figure: *Left: Time series data from the Lorenz system. Right: Reconstructed attractor*

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Two Parameters

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- The time delay τ.

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- The embedding dimension d_E.

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- The embedding dimension d_E.
- Various methods to find these parameters.

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Two Parameters

- The time delay τ.
- The embedding dimension d_E.
- Various methods to find these parameters.
- With no limitations to the general applicability of our approach, we're using $\tau = 1$ and $d_E = 3$ for non-model data.

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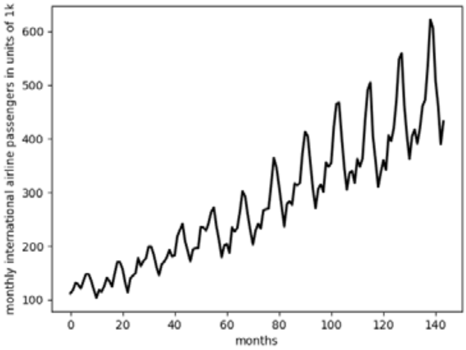
Stochastic Interpolation and Reconstructed Phase Spaces

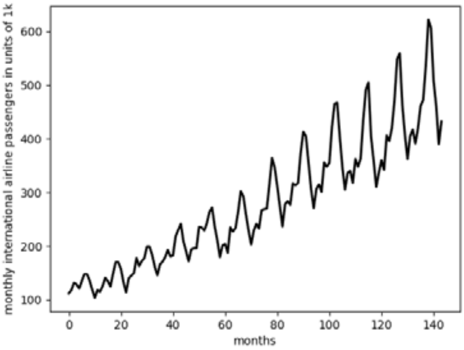
Problem

Problem



monthly international airline passengers in units of 1k
600
500
400
300
200
100
0
20
40
60
80
100
120
140
months

Problem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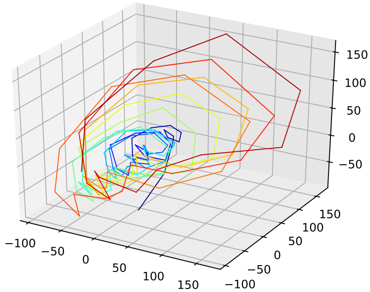
150
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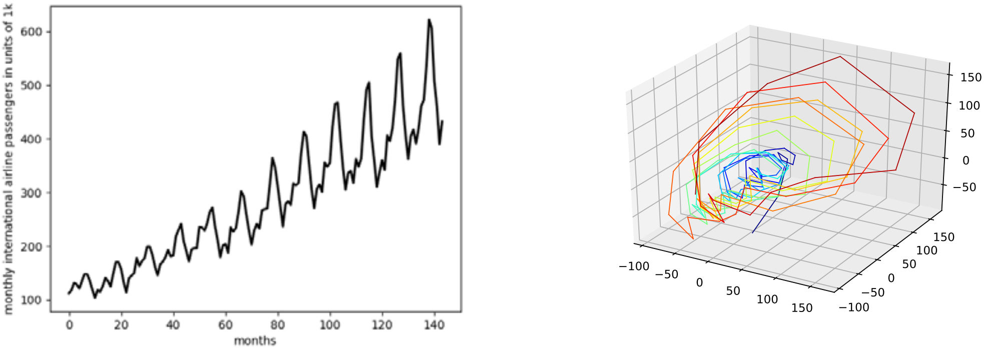

Figure: *Left: Monthly international airline passengers time series; Right: Reconstructed Phase Space*

We want the phase space curve to be smooth.

Idea

[1]: Jan Friedrich, Sebastian Gallon, Alain Pumir, and Rainer Grauer. Stochastic Interpolation of Sparsely Sampled Time Series via Multipoint Fractional Brownian Bridges. *Physical Review Letters*, 125(17):170602, 2020. Publisher: APS

Idea

- We're using multi-point fractional Brownian Bridges to interpolate time series data, [1].

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- The loss/fintess function is the variance of second derivatives along a phase space trajectory.
- PhaSpaSto-interpolation; **pha**se **spa**ce trajectory smoothing **sto**chastic interpolation.

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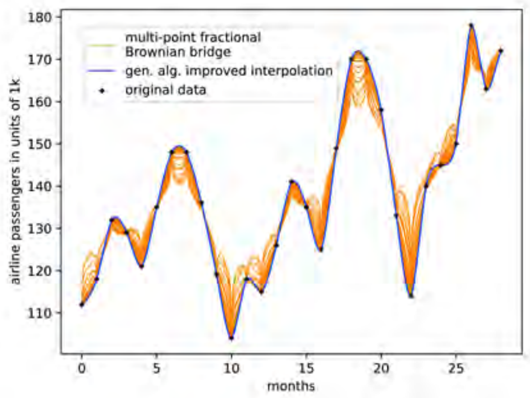

Figure: *Interpolated monthly international airline passengers time series*

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Result

[2]: Sebastian Raubitzek, Thomas Neubauer, Jan Friedrich, and Andreas Rauber. Interpolating strange attractors via fractional brownian bridges. *Entropy*, 24(5), 2022. ISSN 1099-4300. doi: 10.3390/e24050718. URL https://www.mdpi.com/1099-4300/24/5/718

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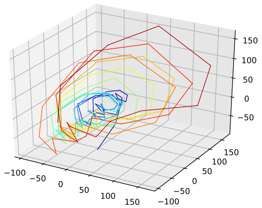

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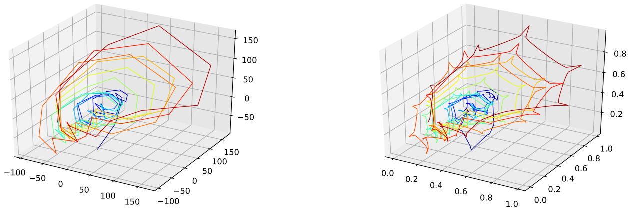

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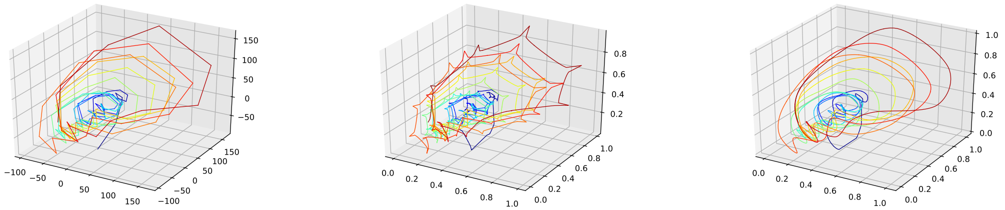

Figure: *Left: Original Phase space portrait; Middle: Phase space portrait population average; Right: Phase space portrait gen. alg. improved. Published in [2].*

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Reconstructed Phase Spaces and Randomly Parameterized Neural Networks

Problem

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- Train and test fits do not guarantee that a neural network can autoregressively predict time series.

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- How to tell if a neural network is capable of reproducing the behaviour by observing the loss?
  - Can't say.

Problem

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Problem

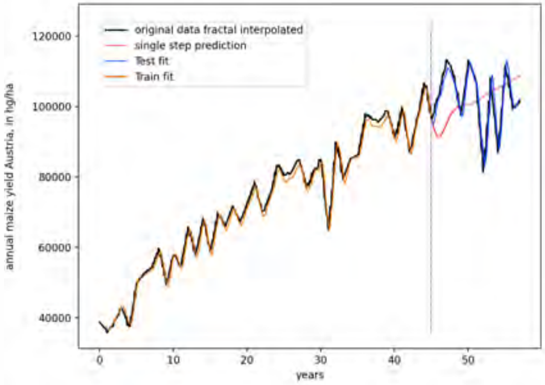

Figure: *Plots for stoch. interpolated annual maize yields in Austria data set.*

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- We produce a multitude, e.g., 500 autoregressive predictions using randomly parameterized neural networks.

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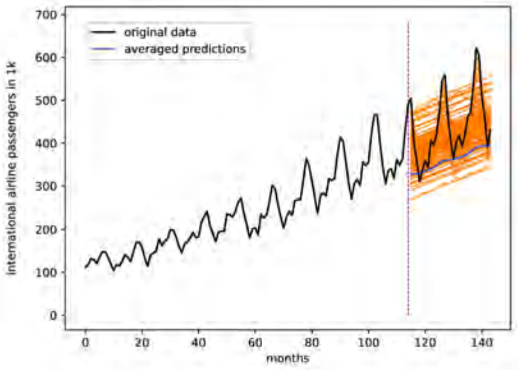
- We filter these predictions based on their reconstructed phase space smoothness.
- We do this for the original and for the interpolated time series. (PhaSpaSto-interpolation)

Idea/Result

Idea/Result



original data
averaged predictions
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months

Idea/Result

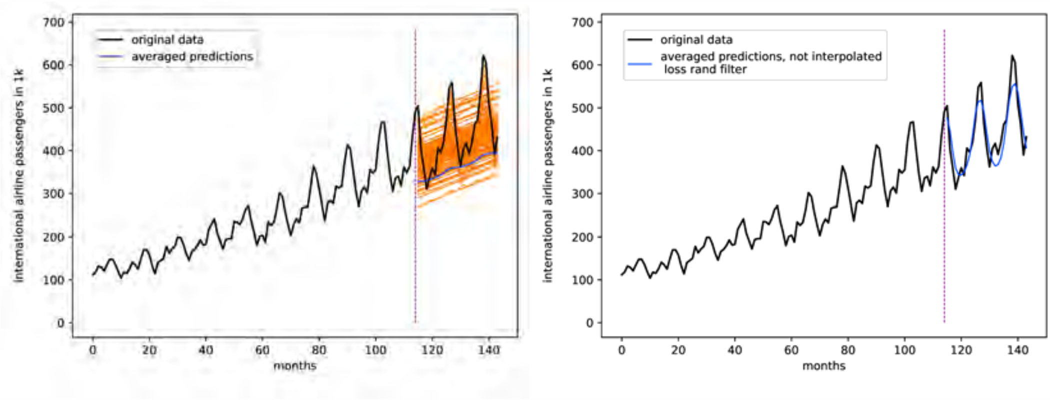

Figure: *Left: non-interpolated unfiltered time series; Right: non-interpolated, filtered time series*

Idea/Result

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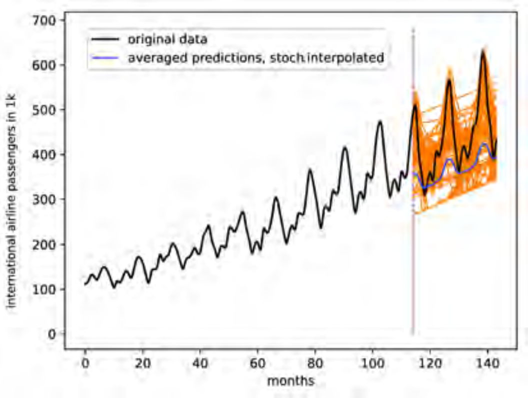

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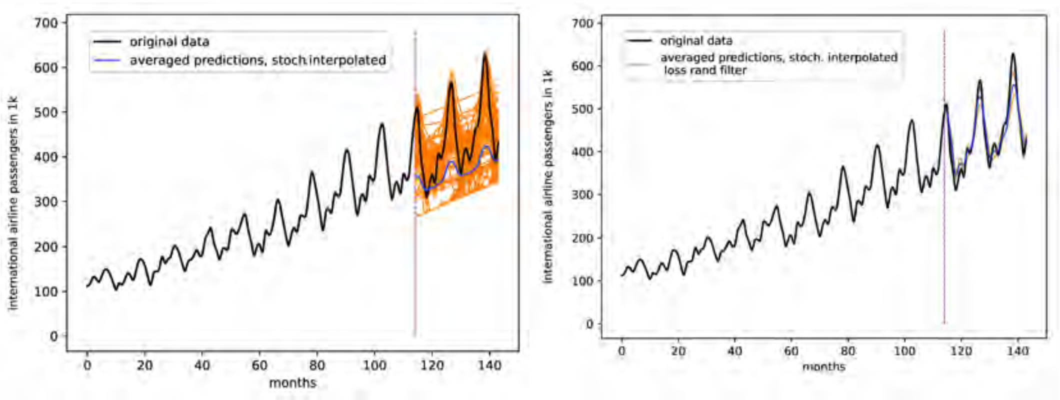

Figure: *Left: stoch. interpolated, 5 interpolation points, unfiltered; Right: stoch. interpolated, 5 interpolation points, filtered*

Idea/Result

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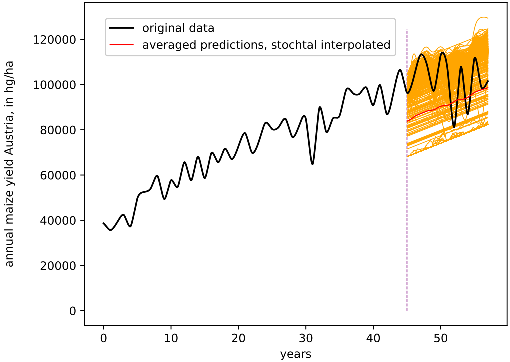
Idea/Result



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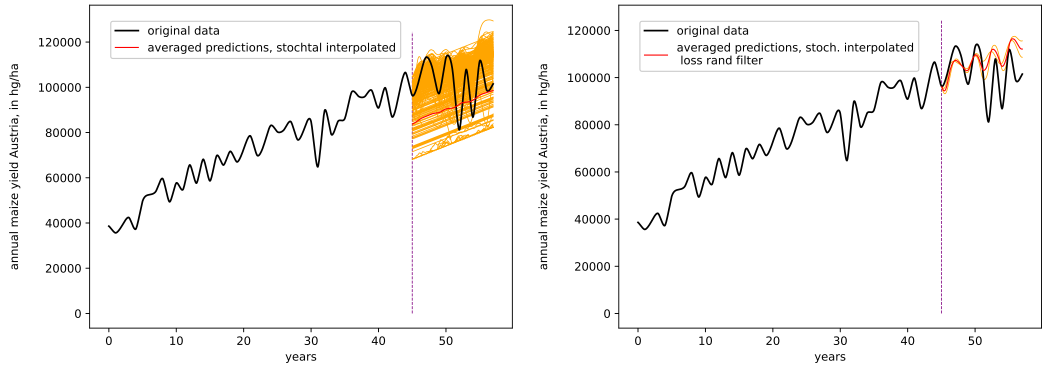

Figure: *Left: stoch. interpolated, 9 interpolation points, unfiltered; Right: stoch. interpolated, 9 interpolation points, filtered*

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Summary

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[2]: Sebastian Raubitzek, Thomas Neubauer, Jan Friedrich, and Andreas Rauber. Interpolating strange attractors via fractional brownian bridges. *Entropy*, 24(5), 2022. ISSN 1099-4300. doi: 10.3390/e24050718. URL `https://www.mdpi.com/1099-4300/24/5/718`
[3]: Sebastian Raubitzek and Thomas Neubauer. Taming the Chaos in Neural Network Time Series Predictions. *Entropy*, 23(11), 2021. ISSN 1099-4300. doi: 10.3390/e23111424. URL `https://www.mdpi.com/1099-4300/23/11/1424`

Summary

- 2 Ideas to combine reconstrcuted Phase Spaces with neural network time series forecasts.

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Summary

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- PhaSpaSto-Interpolation, [2].
- Randomly parameterized neural networks and prediction filters, [3].

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What you should remember from this talk:

- Reconstructed phase spaces and neural network time series forecasts can be combined.
- The variance of second derivatives along a reconstructed phase space curve is a valuable tool for time series analysis. I.e. smoothness.
- Filtering predictions can solve the problem of parameterizing neural networks through randomly parameterizing neural networks.

Thank you for your attention!

[1] Jan Friedrich, Sebastian Gallon, Alain Pumir, and Rainer Grauer. Stochastic Interpolation of Sparsely Sampled Time Series via Multipoint Fractional Brownian Bridges. *Physical Review Letters*, 125(17):170602, 2020. Publisher: APS.

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